MODULE 3

1. How a single perceptron can be used to represent the Boolean functions such as AND, OR

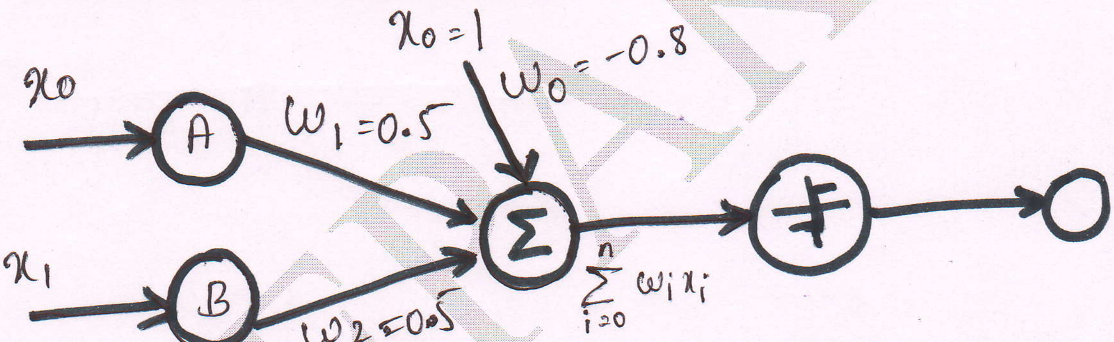
⇒ Boolean function AND

A	B	A ∧ B
0	0	0
0	1	0
1	0	0
1	1	1

* Set $w_0 = -0.8$

$w_1 = 0.5$

$w_2 = 0.5$



$$O(x_1 \ldots x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

1) if $\underset{x_1}{A=0}$ & $\underset{x_2}{B=0}$ ⇒ $\cancel{0*0}$ $-0.8 + (0.5*0) + (0.5*0) =$

$= -0.8 < 0$ So, output $= 0$

2) if A=0 & B=1 ⇒ $-0.8 + (0.5*0) + (0.5*1)$

$= -0.3 < 0$ So, output $= 0$

3) if A=1 & B=1 ⇒ $-0.8 + (0.5*1) + (0.5*0) = -0.3 < 0$

Output = 0

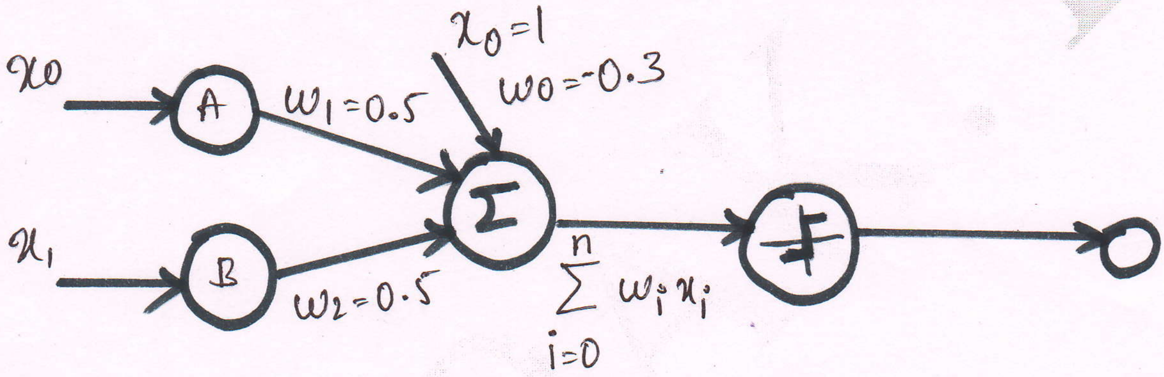
4) if A=1 & B=1 ⇒ $-0.8 + (0.5*1) + (0.5*1) = 0.2 > 0$

output = 1

⇒ Boolean function OR

A	B	A V B
0	0	0
0	1	1
1	0	1
1	1	1

* Set $w_0 = -0.3$
$w_1 = 0.5$
$w_2 = 0.5$



i) A = 0 B = 0 ⇒ $-0.3 + (0.5 * 0) + (0.5 * 0)$
$= -0.3 < 0$ so output = 0

2) A = 0 B = 1 ⇒ $-0.3 + (0.5 * 0) + (0.5 * 1)$
$= 0.2 > 0$ so output = 1

3) A = 1 B = 0 ⇒ $-0.3 + (0.5 * 1) + (0.5 * 0)$
$= 0.2 > 0$ so output = 1

4) A = 1 B = 1 ⇒ $-0.3 + (0.5 * 1) + (0.5 * 0)$
$= 0.7 > 0$ So output = 1

2. (a) Design a two-input perceptron that implements the boolean function A Λ ¬ B. (b) Design a two-layer network of perceptron's that implements A XOR B.

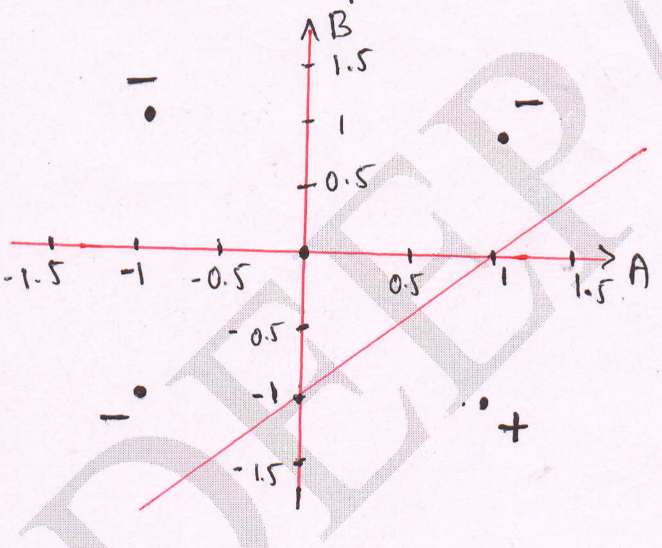
⟹

a) The perceptron has two input A, B and constant 1

A	B	¬B	A Λ ¬B
0 (-1)	0 (-1)	1	0 (-1)
0 (-1)	1	0 (-1)	0 (-1)
1	0 (-1)	1	1
1	1	0	0 (-1)

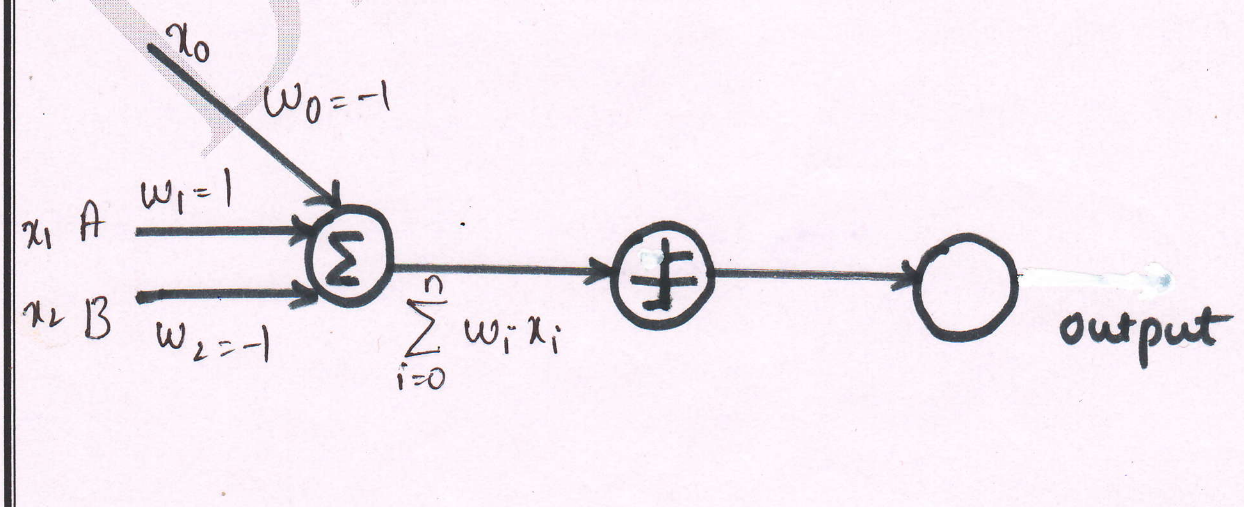
The values of A & B are 1 (true) or -1 or 0 for false.

Decision surfaces



* The line crosses the A axis at 1 and B axis at -1

* The weights are
$w_0 = -1$
$w_1 = 1$ $w_2 = -1$.



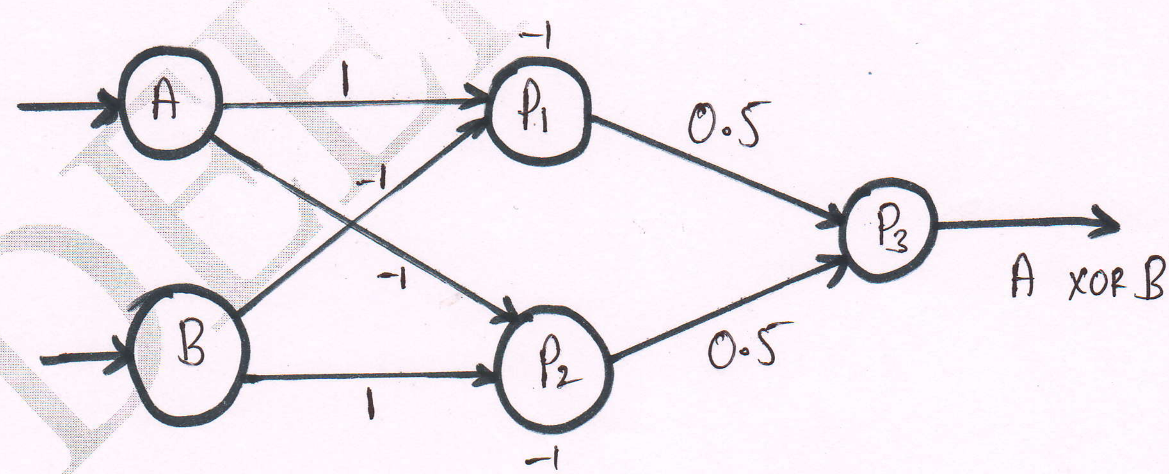
⟹ b) A XOR B cannot be calculated by a single perceptron, so build a two-layer network of perceptrons

* Express A XOR B in terms of other logical connectives

$$A \text{ XOR } B = (A \wedge \neg B) \vee (\neg A \wedge B)$$

* Define the perceptron P_1 and P_2 for $(A \wedge \neg B)$, $(\neg A \wedge B)$

* Composing the outputs of P_1 & P_2 into a perceptron P_3 that implements $O(P_1) \vee O(P_2)$



3. Consider two perceptrons defined by the threshold expression $w_0 + w_1x_1 + w_2x_2 > 0$.

Perceptron A has weight values

$w_0 = 1, w_1=2, w_2=1$

and perceptron B has the weight values

$w_0 = 0, w_1=2, w_2=1$

True or false? Perceptron A is more-general than perceptron B.

Solution

True, Perceptron A is more-general than Perceptron B.

$\Rightarrow O(x_1 \dots x_n) = w_0x_0 + w_1x_1 + w_2x_2 + \dots + w_nx_n$

$\Rightarrow B(\langle x_1, x_2 \rangle) = 1$ & $w_0 = 0, w_1 = 2, w_2 = 1$

$0 + 2x_1 + x_2 > 0 \Rightarrow \underline{0 + 2 + 1 > 0}$

where, x_0 is constant which is equal to 1 i.e., $x_0 = 1$

$\Rightarrow A(\langle x_1, x_2 \rangle) = 1$ & $w_0 = 1, w_1 = 2, w_2 = 1$

$1 + 2x_1 + x_2 > 0 \Rightarrow \underline{1 + 2 + 1 > 0}$

Here, Perceptron A is more general than perceptron B because every instance of x_1 & x_2 that satisfies perceptron B also satisfies perceptron A.